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A HUMAN LOWER LIMB KINEMATIC ANALYSIS

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Abstract: In this a human lower limb kinematic analysis of a 4 years old child for walking case is presented. The research aim is to determine positions, velocities and accelerations of human lower limb joints. The used method has a flexible character easy to implement on computer and assures an interface for a dynamic analysis. The obtained results are useful in the sight of human lower limb prostheses and orthotics mechanisms design and concept.

1. INTRODUCTION

Different methods for study the human lower limb cinematic were developed by specialists and researchers [1], [2], [3], [4].

The kinematic methods aim is to study different motion types of the human lower limb in order to improve the athletes' performances or to design new human lower limb prosthesis. In this case children locomotion system will be analyzed in order to obtain and validate data useful for orthotics design.

The kinematic analysis of a mechanical model consists in solving two important problems: kinematic direct problem and inverse problem.

In the kinematic direct problem's frame, the displacements from kinematic joint are known, and it will be determined the positions – orientation, speeds and accelerations of the mechanism elements or some characteristic points onto the analyzed mechanism.

In the kinematic inverse problem, the parameters for some characteristic points motion are known, and it will be determined the parameters of the kinematic joints relative motions.

With these, in the kinematic analysis context, we identify many problems such as: positional problem; speed problem; accelerations problem.

Each of these problems presents a direct or inverse aspect.

2. HUMAN LOWER LIMB KINEMATIC ANALYSIS

The method used in this paper has a flexible character and assures an interface for dynamic analysis especially for finite element modeling of spatial and planar mobile mechanical systems [7], [8], [10].

For the kinematic analysis the model presented in figure 1, will be considered. The kinematic model analysis will be performed only for walking activity, for a single gait. The kinematic parameters variation laws were obtained by processing with the MAPLE software aid the mathematical models which are defining the human lower limb experimentally analysis.

From a structural viewpoint, the cinematic chain it consists in 8 rotation joints.

The $\bar{r}_i$ position vectors in the T_{i-1} reference coordinate system are:

$$
\begin{aligned}
&r_1 = [0, L_{cx}, 0]^T_{cx}; r_3 = [0, 0, -L_2]^T_2; r_2 = [0, L_1, 0]^T_1; r_4 = [0, -L_3, 0]^T_3;\\
&r_5 = [0, 0, -L_4]^T_4; r_6 = [0, -L_5, 0]^T_5; r_7 = [0, L_6, 0]^T_6; r_8 = [0, -L_7, 0]^T_7;\\
&S_8 = [0, -L_8, 0]^T_8.
\end{aligned} \tag{1.1}
$$

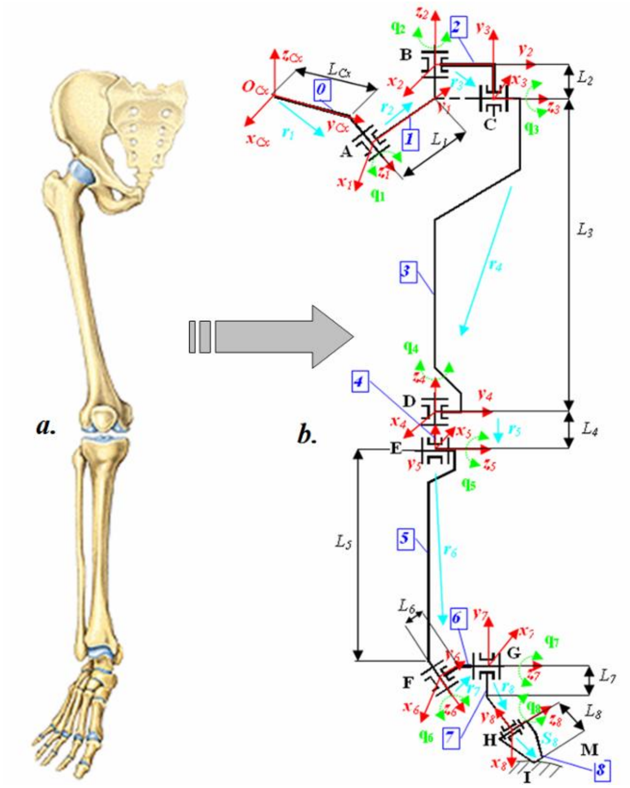

Figure 1. Human lower limb kinematic scheme: a – anatomical model; b – kinematic model

The connectivity order will be: **C_x - 1 - 2 - 3 - 4 - 5 - 6 - 7 – 8.**

1.1. POSITION CALCULUS

The position vectors are:

$$
\begin{aligned}
&\overrightarrow{r_1} = \{r_1^x, r_1^y, r_1^z\} = \{r_1\}^T \cdot \{\overrightarrow{W_{cx}}\}\ \overrightarrow{r_2} = \{r_2^x, r_2^y, r_2^z\} = \{r_2\}^T \cdot \{\overrightarrow{W_1}\}\ \overrightarrow{r_3} = \{r_3^x, r_3^y, r_3^z\} = \{r_3\}^T \cdot \{\overrightarrow{W_2}\}\\
&\overrightarrow{r_4} = \{r_4^x, r_4^y, r_4^z\} = \{r_4\}^T \cdot \{\overrightarrow{W_3}\}\ \overrightarrow{r_5} = \{r_5^x, r_5^y, r_5^z\} = \{r_5\}^T \cdot \{\overrightarrow{W_4}\}\ \overrightarrow{r_6} = \{r_6^x, r_6^y, r_6^z\} = \{r_6\}^T \cdot \{\overrightarrow{W_5}\}\\
&\overrightarrow{r_7} = \{r_7^x, r_7^y, r_7^z\} = \{r_7\}^T \cdot \{\overrightarrow{W_8}\}\ \overrightarrow{r_8} = \{r_8^x, r_8^y, r_8^z\} = \{r_8\}^T \cdot \{\overrightarrow{W_7}\}\ \overrightarrow{S_8} = \{S_8^x, S_8^y, S_8^z\} = \{S_8\}^T \cdot \{\overrightarrow{W_8}\}
\end{aligned} \tag{1.2}
$$

Where:

$$
\begin{aligned}
&\{r_1\}^T = [0, L_{cx}, 0]_{cx}; \{r_2\}^T = [0, L_1, 0]_1;\ \{r_3\}^T = [0, 0, -L_2]_2; \{r_4\}^T = [0, -L_3, 0]_3;\\
&\{r_5\}^T = [0, 0, -L_4]_4; \{r_6\}^T = [0, -L_5, 0]_5;\ \{r_7\}^T = [0, L_6, 0]_6; \{r_8\}^T = [0, -L_7, 0]_7;\\
&\{S_8\}^T = [0, -L_8, 0]_8.
\end{aligned} \tag{1.3}
$$

The $\overrightarrow{r_M^{Cx}}$ vector, has the following expression:

$$\overrightarrow{r_M^{Cx}} = \vec{r_1} + \vec{r_2} + \vec{r_3} + \vec{r_4} + \vec{r_5} + \vec{r_6} + \vec{r_7} + \vec{r_8} + \overrightarrow{S_8} \tag{1.4}$$

Changing the versors base at crossing from a reference coordinate system to another (introducing the coordinate transformation matrices):

$$\begin{aligned}
&\{\overline{W_1}\} = [A_{Cx1}]\cdot\{\overline{W_{Cx}}\};\ \{\overline{W_2}\} = [A_{12}]\cdot\{\overline{W_1}\} = [A_{Cx2}]\cdot\{\overline{W_{Cx}}\}\\
&\{\overline{W_3}\} = [A_{23}]\cdot\{\overline{W_2}\} = [A_{Cx3}]\cdot\{\overline{W_{Cx}}\};\ \{\overline{W_4}\} = [A_{34}]\cdot\{\overline{W_3}\} = [A_{Cx4}]\cdot\{\overline{W_{Cx}}\};\\
&\{\overline{W_5}\} = [A_{45}]\cdot\{\overline{W_4}\} = [A_{Cx5}]\cdot\{\overline{W_{Cx}}\};\ \{\overline{W_6}\} = [A_{56}]\cdot\{\overline{W_5}\} = [A_{Cx6}]\cdot\{\overline{W_{Cx}}\};\\
&\{\overline{W_7}\} = [A_{67}]\cdot\{\overline{W_6}\} = [A_{Cx7}]\cdot\{\overline{W_{Cx}}\};\ \{\overline{W_8}\} = [A_{78}]\cdot\{\overline{W_7}\} = [A_{Cx8}]\cdot\{\overline{W_{Cx}}\};
\end{aligned} \tag{1.5}$$

By analyzing (1.5) equations we observe that:

$$[A_{Cx2}] = [A_{12}]\cdot[A_{Cx1}] \tag{1.6}$$

$$[A_{Cx3}] = [A_{23}]\cdot[A_{12}]\cdot[A_{Cx1}] \tag{1.7}$$

$$[A_{Cx4}] = [A_{34}]\cdot[A_{23}]\cdot[A_{12}]\cdot[A_{Cx1}] = [A_{34}]\cdot[A_{Cx3}] \tag{1.8}$$

$$[A_{Cx5}] = [A_{45}]\cdot[A_{34}]\cdot[A_{23}]\cdot[A_{12}]\cdot[A_{Cx1}] = [A_{45}]\cdot[A_{Cx4}] \tag{1.9}$$

$$[A_{Cx6}] = [A_{56}]\cdot[A_{45}]\cdot[A_{34}]\cdot[A_{23}]\cdot[A_{12}]\cdot[A_{Cx1}] = [A_{56}]\cdot[A_{Cx5}] \tag{1.10}$$

$$[A_{Cx7}] = [A_{67}]\cdot[A_{56}]\cdot[A_{45}]\cdot[A_{34}]\cdot[A_{23}]\cdot[A_{12}]\cdot[A_{Cx1}] = [A_{67}]\cdot[A_{Cx6}] \tag{1.11}$$

$$[A_{Cx8}] = [A_{78}]\cdot[A_{67}]\cdot[A_{56}]\cdot[A_{45}]\cdot[A_{34}]\cdot[A_{23}]\cdot[A_{12}]\cdot[A_{Cx1}] = [A_{78}]\cdot[A_{Cx7}] \tag{1.12}$$

Based on (1.6)... (1.12) one identify the coordinates transformation matrices for each cinematic joints, with $\alpha_{i,i+1} = 90^\circ$, and $i = \overline{1,8}$.

Point: **A, B, C, D, E, F, G, H and M** positions in rapport with T_{cx} coordinate system, bounded to the coxae bone, will be identified through relations:

$$\left\{\overline{r_A}^{T_{Cx}}\right\} = \{r_1\}^T\cdot\{\overline{W_{Cx}}\} \tag{1.13}$$

$$\left\{\overline{r_B}^{T_{Cx}}\right\} = \{r_1\}^T\cdot\{\overline{W_{Cx}}\} + \{r_2\}^T\cdot[A_{Cx1}]\cdot\{\overline{W_{Cx}}\} \tag{1.14}$$

$$\left\{\overline{r_C}^{T_{Cx}}\right\} = \{r_1\}^T\cdot\{\overline{W_{Cx}}\} + \{r_2\}^T\cdot[A_{Cx1}]\cdot\{\overline{W_{Cx}}\} + \{r_3\}^T\cdot[A_{12}]\cdot[A_{Cx1}]\cdot\{\overline{W_{Cx}}\} \tag{1.15}$$

$$\begin{aligned}
\left\{\overline{r_D}^{T_{Cx}}\right\} &= \{r_1\}^T\cdot\{\overline{W_{Cx}}\} + \{r_2\}^T\cdot[A_{Cx1}]\cdot\{\overline{W_{Cx}}\} + \{r_3\}^T\cdot[A_{12}]\cdot[A_{Cx1}]\cdot\{\overline{W_{Cx}}\} +\\
&+ \{r_4\}^T\cdot[A_{23}]\cdot[A_{12}]\cdot[A_{Cx1}]\cdot\{\overline{W_{Cx}}\}
\end{aligned} \tag{1.16}$$

$$\begin{aligned}
\left\{\overline{r_E}^{T_{Cx}}\right\} &= \{r_1\}^T\cdot\{\overline{W_{Cx}}\} + \{r_2\}^T\cdot[A_{Cx1}]\cdot\{\overline{W_{Cx}}\} + \{r_3\}^T\cdot[A_{12}]\cdot[A_{Cx1}]\cdot\{\overline{W_{Cx}}\} + \{r_4\}^T\cdot[A_{23}]\cdot[A_{12}]\cdot\\
&\cdot[A_{Cx1}]\cdot\{\overline{W_{Cx}}\} + \{r_5\}^T\cdot[A_{34}]\cdot[A_{23}]\cdot[A_{12}]\cdot[A_{Cx1}]\cdot\{\overline{W_{Cx}}\}
\end{aligned} \tag{1.17}$$

$$\begin{aligned}
\left\{\overline{r_F}^{T_{Cx}}\right\} &= \{r_1\}^T\cdot\{\overline{W_{Cx}}\} + \{r_2\}^T\cdot[A_{Cx1}]\cdot\{\overline{W_{Cx}}\} + \{r_3\}^T\cdot[A_{12}]\cdot[A_{Cx1}]\cdot\{\overline{W_{Cx}}\} + \{r_4\}^T\cdot[A_{23}]\cdot[A_{12}]\cdot\\
&\cdot[A_{Cx1}]\cdot\{\overline{W_{Cx}}\} + \{r_5\}^T\cdot[A_{34}]\cdot[A_{23}]\cdot[A_{12}]\cdot[A_{Cx1}]\cdot\{\overline{W_{Cx}}\} + \{r_6\}^T\cdot[A_{45}]\cdot[A_{34}]\cdot[A_{23}]\cdot[A_{12}]\cdot[A_{Cx1}]\cdot\{\overline{W_{Cx}}\}
\end{aligned} \tag{1.18}$$

$$\begin{aligned}
\left\{\overline{r_G}^{T_{Cx}}\right\} &= \{r_1\}^T\cdot\{\overline{W_{Cx}}\} + \{r_2\}^T\cdot[A_{Cx1}]\cdot\{\overline{W_{Cx}}\} + \{r_3\}^T\cdot[A_{12}]\cdot[A_{Cx1}]\cdot\{\overline{W_{Cx}}\} + \{r_4\}^T\cdot[A_{23}]\cdot[A_{12}]\cdot[A_{Cx1}]\cdot\\
&\cdot\{\overline{W_{Cx}}\} + \{r_5\}^T\cdot[A_{34}]\cdot[A_{23}]\cdot[A_{12}]\cdot[A_{Cx1}]\cdot\{\overline{W_{Cx}}\} + \{r_6\}^T\cdot[A_{45}]\cdot[A_{34}]\cdot[A_{23}]\cdot[A_{12}]\cdot[A_{Cx1}]\cdot\{\overline{W_{Cx}}\} +\\
&+ \{r_7\}^T\cdot[A_{56}]\cdot[A_{45}]\cdot[A_{34}]\cdot[A_{23}]\cdot[A_{12}]\cdot[A_{Cx1}]\cdot\{\overline{W_{Cx}}\}
\end{aligned} \tag{1.19}$$

$$\begin{aligned}\left\{\overline{r_H}^{T_{Cx}}\right\} &= \{r_1\}^T \cdot \{\overline{W}_{Cx}\} + \{r_2\}^T \cdot [A_{Cx1}] \cdot \{\overline{W}_{Cx}\} + \{r_3\}^T \cdot [A_{12}] \cdot [A_{Cx1}] \cdot \{\overline{W}_{Cx}\} + \{r_4\}^T \cdot [A_{23}] \cdot [A_{12}] \cdot \\ &\cdot [A_{Cx1}] \cdot \{\overline{W}_{Cx}\} + \{r_5\}^T \cdot [A_{34}] \cdot [A_{23}] \cdot [A_{12}] \cdot [A_{Cx1}] \cdot \{\overline{W}_{Cx}\} + \{r_6\}^T \cdot [A_{45}] \cdot [A_{34}] \cdot [A_{23}] \cdot [A_{12}] \cdot [A_{Cx1}] \cdot \\ &\cdot \{\overline{W}_{Cx}\} + \{r_7\}^T \cdot [A_{56}] \cdot [A_{45}] \cdot [A_{34}] \cdot [A_{23}] \cdot [A_{12}] \cdot [A_{Cx1}] \cdot \{\overline{W}_{Cx}\} + \{r_8\}^T \cdot [A_{67}] \cdot [A_{56}] \cdot [A_{45}] \cdot [A_{34}] \cdot \\ &\cdot [A_{23}] \cdot [A_{12}] \cdot [A_{Cx1}] \cdot \{\overline{W}_{Cx}\}\end{aligned} \tag{1.20}$$

$$\begin{aligned}\left\{\overline{r_M}^{T_{Cx}}\right\} &= \{r_1\}^T \cdot \{\overline{W}_{Cx}\} + \{r_2\}^T \cdot [A_{Cx1}] \cdot \{\overline{W}_{Cx}\} + \{r_3\}^T \cdot [A_{12}] \cdot [A_{Cx1}] \cdot \{\overline{W}_{Cx}\} + \{r_4\}^T \cdot [A_{23}] \cdot [A_{12}] \cdot \\ &\cdot [A_{Cx1}] \cdot \{\overline{W}_{Cx}\} + \{r_5\}^T \cdot [A_{34}] \cdot [A_{23}] \cdot [A_{12}] \cdot [A_{Cx1}] \cdot \{\overline{W}_{Cx}\} + \{r_6\}^T \cdot [A_{45}] \cdot [A_{34}] \cdot [A_{23}] \cdot [A_{12}] \cdot [A_{Cx1}] \cdot \\ &\cdot \{\overline{W}_{Cx}\} + \{r_7\}^T \cdot [A_{56}] \cdot [A_{45}] \cdot [A_{34}] \cdot [A_{23}] \cdot [A_{12}] \cdot [A_{Cx1}] \cdot \{\overline{W}_{Cx}\} + \{r_8\}^T \cdot [A_{67}] \cdot [A_{56}] \cdot [A_{45}] \cdot [A_{34}] \cdot \\ &\cdot [A_{23}] \cdot [A_{12}] \cdot [A_{Cx1}] \cdot \{\overline{W}_{Cx}\} + \{S_8\}^T \cdot [A_{78}] \cdot [A_{67}] \cdot [A_{56}] \cdot [A_{45}] \cdot [A_{34}] \cdot [A_{23}] \cdot [A_{12}] \cdot [A_{Cx1}] \cdot \{\overline{W}_{Cx}\}\end{aligned} \tag{1.21}$$

1.2. SPEED CALCULUS

We follow to determine the **M** point speed in rapport with T_{cx} reference system. For this we differentiate successively the (1.13) ... (1.21) relations, but for achieve this calculus is necessary to build the anti symmetric matrices for each joint, like this form:

$$\left[\tilde{\omega}_{Cxi}\right] = \begin{bmatrix} 0 & \omega_{Cxi} & -\omega_{Cxi} \\ -\omega_{Cxi} & 0 & \omega_{Cxi} \\ \omega_{Cxj} & -\omega_{Cxj} & 0 \end{bmatrix}, \text{ with } i, j = \overline{1,8} \tag{1.22}$$

For this:

$$\begin{aligned}&\left[\dot{A}_{Cx1}\right] = \left[\tilde{\omega}_{Cx1}\right] \cdot \left[A_{Cx1}\right]; \ \left[\dot{A}_{Cx2}\right] = \left[\tilde{\omega}_{12}\right] \cdot \left[A_{Cx3}\right]; \ \left[\dot{A}_{Cx3}\right] = \left[\tilde{\omega}_{23}\right] \cdot \left[A_{Cx3}\right]; \\ &\left[\dot{A}_{Cx4}\right] = \left[\tilde{\omega}_{34}\right] \cdot \left[A_{Cx4}\right]; \ \left[\dot{A}_{Cx5}\right] = \left[\tilde{\omega}_{45}\right] \cdot \left[A_{Cx5}\right]; \ \left[\dot{A}_{Cx6}\right] = \left[\tilde{\omega}_{56}\right] \cdot \left[A_{Cx7}\right]; \\ &\left[\dot{A}_{Cx7}\right] = \left[\tilde{\omega}_{67}\right] \cdot \left[A_{Cx7}\right]; \ \left[\dot{A}_{Cx8}\right] = \left[\tilde{\omega}_{78}\right] \cdot \left[A_{Cx8}\right].\end{aligned} \tag{1.23}$$

For each: **A, B, C, D, E, F, G, H** and **M** point we obtain:

$$\left\{\overline{v_A}^{Tcx}\right\} = 0 \tag{1.24}$$

$$\left\{\overline{v_B}^{Tcx}\right\} = 0 + \{r_2\}^T \cdot \left[\tilde{\omega}_{Cx1}\right] \cdot [A_{Cx1}] \cdot \{\overline{W}_{Cx}\} \tag{1.25}$$

$$\left\{\overline{v_C}^{Tcx}\right\} = 0 + \{r_2\}^T \cdot \left[\tilde{\omega}_{Cx1}\right] \cdot [A_{Cx1}] \cdot \{\overline{W}_{Cx}\} + \{r_3\}^T \cdot \left[\tilde{\omega}_{12}\right] \cdot [A_{Cx2}] \cdot \{\overline{W}_{Cx}\} \tag{1.26}$$

$$\left\{\overline{v_D}^{Tcx}\right\} = 0 + \{r_2\}^T \cdot \left[\tilde{\omega}_{Cx1}\right] \cdot [A_{Cx1}] \cdot \{\overline{W}_{Cx}\} + \{r_3\}^T \cdot \left[\tilde{\omega}_{12}\right] \cdot [A_{Cx2}] \cdot \{\overline{W}_{Cx}\} + \{r_4\}^T \cdot \left[\tilde{\omega}_{23}\right] \cdot [A_{Cx3}] \cdot \{\overline{W}_{Cx}\} \tag{1.27}$$

$$\begin{aligned}\left\{\overline{v_E}^{Tcx}\right\} &= 0 + \{r_2\}^T \cdot \left[\tilde{\omega}_{Cx1}\right] \cdot [A_{Cx1}] \cdot \{\overline{W}_{Cx}\} + \{r_3\}^T \cdot \left[\tilde{\omega}_{12}\right] \cdot [A_{Cx2}] \cdot \{\overline{W}_{Cx}\} + \{r_4\}^T \cdot \left[\tilde{\omega}_{23}\right] \cdot \\ &\cdot [A_{Cx3}] \cdot \{\overline{W}_{Cx}\} + \{r_5\}^T \cdot \left[\tilde{\omega}_{34}\right] \cdot [A_{Cx4}] \cdot \{\overline{W}_{Cx}\}\end{aligned} \tag{1.28}$$

$$\left\{\overline{v_F}^{Tcx}\right\} = 0 + \{r_2\}^T \cdot \left[\tilde{\omega}_{Cx1}\right] \cdot \left[A_{Cx1}\right] \cdot \left\{\overline{W}_{Cx}\right\} + \{r_3\}^T \cdot \left[\tilde{\omega}_{12}\right] \cdot \left[A_{Cx2}\right] \cdot \left\{\overline{W}_{Cx}\right\} + \{r_4\}^T \cdot \left[\tilde{\omega}_{23}\right] \cdot \\ \cdot \left[A_{Cx3}\right] \cdot \left\{\overline{W}_{Cx}\right\} + \{r_5\}^T \cdot \left[\tilde{\omega}_{34}\right] \cdot \left[A_{Cx4}\right] \cdot \left\{\overline{W}_{Cx}\right\} + \{r_6\}^T \cdot \left[\tilde{\omega}_{45}\right] \cdot \left[A_{Cx5}\right] \cdot \left\{\overline{W}_{Cx}\right\} \quad (1.29)$$

$$\left\{\overline{v_G}^{Tcx}\right\} = 0 + \{r_2\}^T \cdot \left[\tilde{\omega}_{Cx1}\right] \cdot \left[A_{Cx1}\right] \cdot \left\{\overline{W}_{Cx}\right\} + \{r_3\}^T \cdot \left[\tilde{\omega}_{12}\right] \cdot \left[A_{Cx2}\right] \cdot \left\{\overline{W}_{Cx}\right\} + \{r_4\}^T \cdot \left[\tilde{\omega}_{23}\right] \cdot \left[A_{Cx3}\right] \cdot \\ \cdot \left\{\overline{W}_{Cx}\right\} + \{r_5\}^T \cdot \left[\tilde{\omega}_{34}\right] \cdot \left[A_{Cx4}\right] \cdot \left\{\overline{W}_{Cx}\right\} + \{r_6\}^T \cdot \left[\tilde{\omega}_{45}\right] \cdot \left[A_{Cx5}\right] \cdot \left\{\overline{W}_{Cx}\right\} + \{r_7\}^T \cdot \left[\tilde{\omega}_{56}\right] \cdot \left[A_{Cx6}\right] \cdot \left\{\overline{W}_{Cx}\right\} \quad (1.30)$$

$$\left\{\overline{v_H}^{Tcx}\right\} = 0 + \{r_2\}^T \cdot \left[\tilde{\omega}_{Cx1}\right] \cdot \left[A_{Cx1}\right] \cdot \left\{\overline{W}_{Cx}\right\} + \{r_3\}^T \cdot \left[\tilde{\omega}_{12}\right] \cdot \left[A_{Cx2}\right] \cdot \left\{\overline{W}_{Cx}\right\} + \{r_4\}^T \cdot \left[\tilde{\omega}_{23}\right] \cdot \\ \cdot \left[A_{Cx3}\right] \cdot \left\{\overline{W}_{Cx}\right\} + \{r_5\}^T \cdot \left[\tilde{\omega}_{34}\right] \cdot \left[A_{Cx4}\right] \cdot \left\{\overline{W}_{Cx}\right\} + \{r_6\}^T \cdot \left[\tilde{\omega}_{45}\right] \cdot \left[A_{Cx5}\right] \cdot \left\{\overline{W}_{Cx}\right\} + \{r_7\}^T \cdot \left[\tilde{\omega}_{56}\right] \cdot \quad (1.31) \\ \cdot \left[A_{Cx6}\right] \cdot \left\{\overline{W}_{Cx}\right\} + \{r_8\}^T \cdot \left[\tilde{\omega}_{67}\right] \cdot \left[A_{Cx7}\right] \cdot \left\{\overline{W}_{Cx}\right\}$$

$$\left\{\overline{v_M}^{Tcx}\right\} = 0 + \{r_2\}^T \cdot \left[\tilde{\omega}_{Cx1}\right] \cdot \left[A_{Cx1}\right] \cdot \left\{\overline{W}_{Cx}\right\} + \{r_3\}^T \cdot \left[\tilde{\omega}_{12}\right] \cdot \left[A_{Cx2}\right] \cdot \left\{\overline{W}_{Cx}\right\} + \{r_4\}^T \cdot \left[\tilde{\omega}_{23}\right] \cdot \\ \cdot \left[A_{Cx3}\right] \cdot \left\{\overline{W}_{Cx}\right\} + \{r_5\}^T \cdot \left[\tilde{\omega}_{34}\right] \cdot \left[A_{Cx4}\right] \cdot \left\{\overline{W}_{Cx}\right\} + \{r_6\}^T \cdot \left[\tilde{\omega}_{45}\right] \cdot \left[A_{Cx5}\right] \cdot \left\{\overline{W}_{Cx}\right\} + \{r_7\}^T \cdot \left[\tilde{\omega}_{56}\right] \cdot \quad (1.32) \\ \cdot \left[A_{Cx6}\right] \cdot \left\{\overline{W}_{Cx}\right\} + \{r_8\}^T \cdot \left[\tilde{\omega}_{67}\right] \cdot \left[A_{Cx7}\right] \cdot \left\{\overline{W}_{Cx}\right\} + \{S_8\}^T \cdot \left[\tilde{\omega}_{78}\right] \cdot \left[A_{Cx8}\right] \cdot \left\{\overline{W}_{Cx}\right\}$$

1.3. ACCELERATION CALCULUS

These will be obtained by differentiating successively the (1.24) ... (1.32). For **A,** and **B**, we will obtain the accelerations from (1.33) and (1.34) equations. Similarly, we obtain the accelerations of the following points: **C, D, E, F, G, H**. The **M** point acceleration is given by (1.35).

$$\left\{\overline{a_A}^{Tcx}\right\} = 0 \quad (1.33)$$

$$\left\{\overline{a_B}^{Tcx}\right\} = 0 + \{r_2\}^T \cdot \left[\dot{\tilde{\omega}}_{Cx1}\right] \cdot \left[A_{Cx1}\right] \cdot \left\{\overline{W}_{Cx}\right\} + \{r_2\}^T \cdot \left[\tilde{\omega}_{Cx1}\right] \cdot \left[\tilde{\omega}_{Cx1}\right] \cdot \left[A_{Cx1}\right] \cdot \left\{\overline{W}_{Cx}\right\} \quad (1.34)$$

$$
\begin{aligned}
\left\{\overline{a_M}^{Tcx}\right\} = {} & 0+\{r_2\}^T\cdot\left[\dot{\tilde{\omega}}_{Cx1}\right]\cdot\left[A_{Cx1}\right]\cdot\left\{\overline{W}_{Cx}\right\}+\{r_2\}^T\cdot\left[\tilde{\omega}_{Cx1}\right]\cdot\left[\tilde{\omega}_{Cx1}\right]\cdot\left[A_{Cx1}\right]\cdot\left\{\overline{W}_{Cx}\right\}+\\
&+\{r_3\}^T\cdot\left[\dot{\tilde{\omega}}_{12}\right]\cdot\left[A_{Cx2}\right]\cdot\left\{\overline{W}_{Cx}\right\}+\{r_3\}^T\cdot\left[\tilde{\omega}_{12}\right]\cdot\left[\tilde{\omega}_{12}\right]\cdot\left[A_{Cx2}\right]\cdot\left\{\overline{W}_{Cx}\right\}++\{r_4\}^T\cdot\left[\dot{\tilde{\omega}}_{23}\right]\cdot\left[A_{Cx3}\right]\cdot\left\{\overline{W}_{Cx}\right\}+\\
&+\{r_4\}^T\cdot\left[\tilde{\omega}_{23}\right]\cdot\left[\tilde{\omega}_{23}\right]\cdot\left[A_{Cx3}\right]\cdot\left\{\overline{W}_{Cx}\right\}+\{r_5\}^T\cdot\left[\dot{\tilde{\omega}}_{34}\right]\cdot\left[A_{Cx4}\right]\cdot\left\{\overline{W}_{Cx}\right\}+\{r_5\}^T\cdot\left[\tilde{\omega}_{34}\right]\cdot\left[\tilde{\omega}_{34}\right]\cdot\left[A_{Cx4}\right]\cdot\left\{\overline{W}_{Cx}\right\}+\\
&+\{r_6\}^T\cdot\left[\dot{\tilde{\omega}}_{45}\right]\cdot\left[A_{Cx5}\right]\cdot\left\{\overline{W}_{Cx}\right\}+\{r_6\}^T\cdot\left[\tilde{\omega}_{45}\right]\cdot\left[\tilde{\omega}_{45}\right]\cdot\left[A_{Cx5}\right]\cdot\left\{\overline{W}_{Cx}\right\}+\{r_7\}^T\cdot\left[\dot{\tilde{\omega}}_{56}\right]\cdot\left[A_{Cx6}\right]\cdot\left\{\overline{W}_{Cx}\right\}+\\
&+\{r_7\}^T\cdot\left[\tilde{\omega}_{56}\right]\cdot\left[\tilde{\omega}_{56}\right]\cdot\left[A_{Cx6}\right]\cdot\left\{\overline{W}_{Cx}\right\}+\{r_8\}^T\cdot\left[\dot{\tilde{\omega}}_{67}\right]\cdot\left[A_{Cx7}\right]\cdot\left\{\overline{W}_{Cx}\right\}+\{r_8\}^T\cdot\left[\tilde{\omega}_{67}\right]\cdot\left[\tilde{\omega}_{67}\right]\cdot\left[A_{Cx7}\right]\cdot\left\{\overline{W}_{Cx}\right\}+\\
&+\{S_8\}^T\cdot\left[\dot{\tilde{\omega}}_{78}\right]\cdot\left[A_{Cx8}\right]\cdot\left\{\overline{W}_{Cx}\right\}+\{S_8\}^T\cdot\left[\tilde{\omega}_{78}\right]\cdot\left[\tilde{\omega}_{78}\right]\cdot\left[A_{Cx8}\right]\cdot\left\{\overline{W}_{Cx}\right\}
\end{aligned}
\tag{1.35}
$$

3. NUMERICAL PROCESSING

For the kinematic analysis we consider known the geometrical elements. The calculus algorithm was elaborated with the MAPLE software's aid. The geometrical elements dimensions are in millimeters: L_{cx}=110; L_1=5; L_2=5; L_3=200; L_4=5; L_5=225; L_6=5; L_7=65, L_8=45.The generalized coordinate laws are known (q1, q2, q3, q4, q5, q6, q7, q8). The M point displacement components on x,y axes are represented in figures 2 and 3. Similarly speed and acceleration motion laws are obtained.

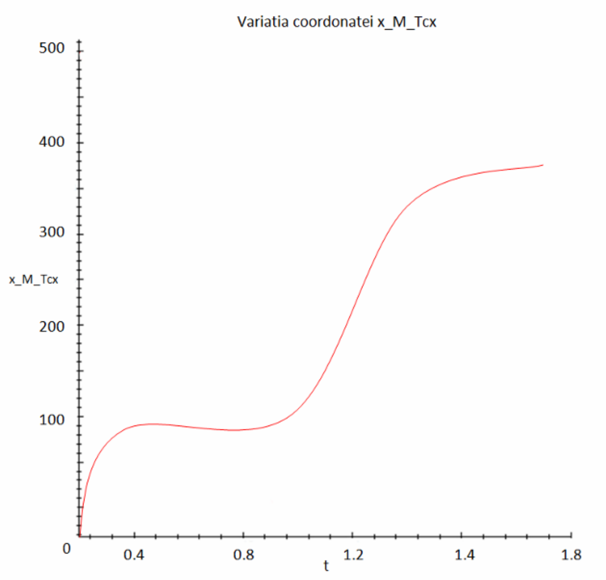

Figure 2. M point component on x axis for walking activity

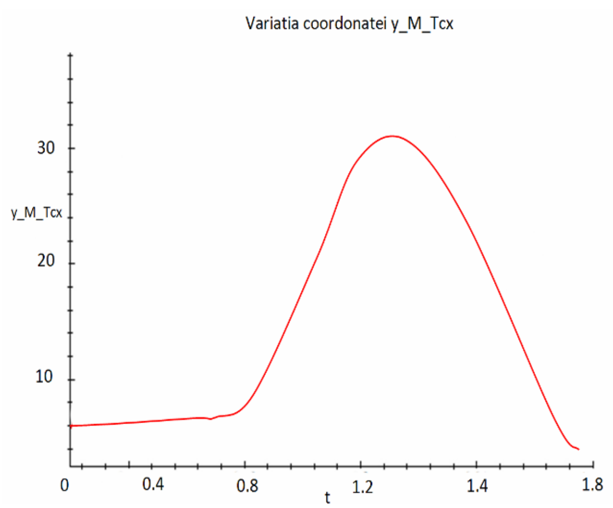

Figure 3. M point component on y axis for walking activity

4. KINEMATIC RESULTS VALIDATION THROUGH EXPERIMENTAL RESEARCH

In order to validate the kinematic results obtained on analytical way, an experimental research was developed on a 4 years old child for walking activity. The equipment used in this research frame was presented in different research articles, and this is called CONTEMPLAS Motion Analysis which exists in Faculty of Mechanics – University of Craiova. With this equipment can determine angular linear variations in 2D or 3D mode. The equipment analysis scheme is presented in figure 4. On the human lower limb one attach a reflexive marker in each joint centre. For the human lower limb experimental analysis 6 markers were attached.

An aspect from the experimental analysis and also the Templo Standard Software is presented in figure 5.

The obtained results are represented through displacements of human lower limb joint centers on x and y-axis. These variations are represented in figure 6 and 7.

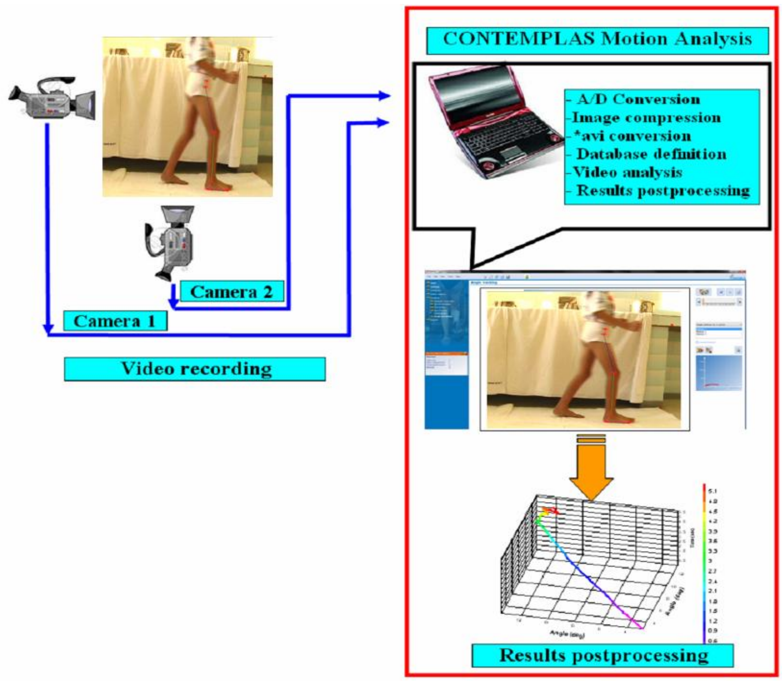

Figure 4. CONTEMPLAS equipment analysis scheme

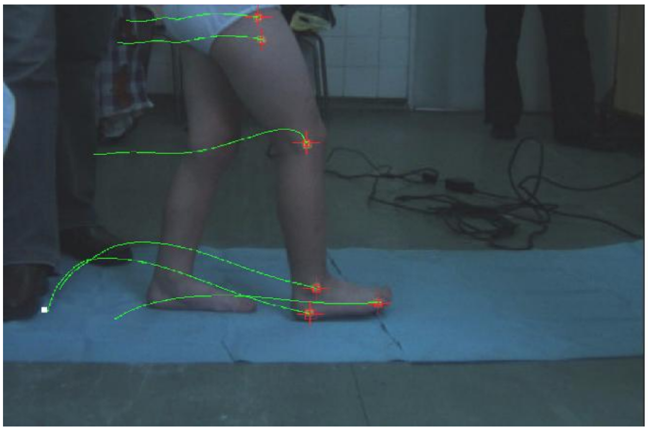

Figure 5. CONTEMPLAS – Templo Standard interface analysis during a single gait

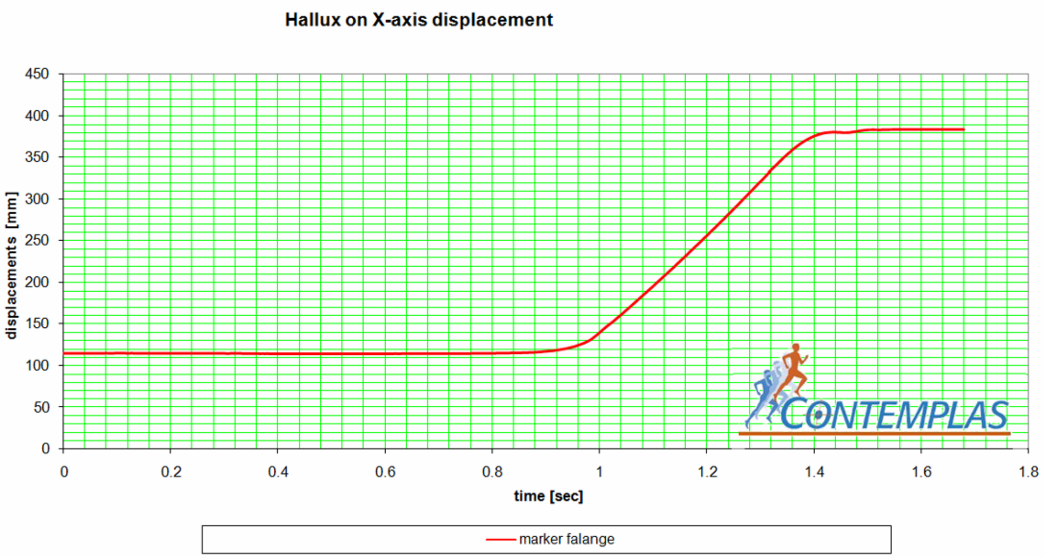

Figure 6. Hallux displacements on X axis generated by the CONTEMPLAS software for walking in case of a 4 years old child

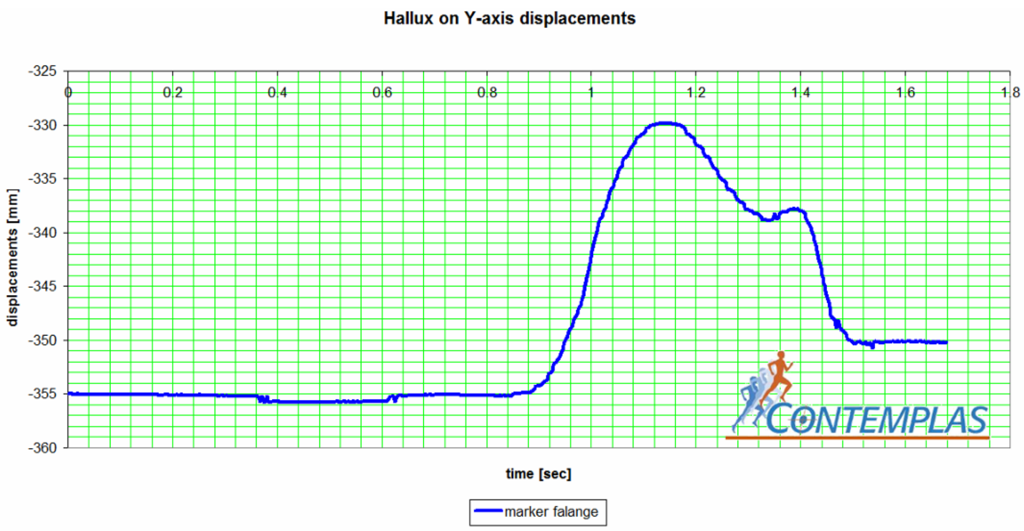

Figure 7. Hallux displacements on X axis generated by the CONTEMPLAS software for walking in case of a 4 years old child

5. CONCLUSIONS

For kinematic modeling we use a method which is based on simple matrices formalism with the possibility to implement on a computer program for direct or inverse kinematic analysis. This method is valid for planar and spatial kinematic mechanisms with possibility to study kinematic parameters in the absolute or relative motion mode. For the mathematical models computing according with the kinematic analysis, an algorithm under MAPLE programming language was elaborated.

A kinematic scheme for the human lower limb equivalent mechanism was developed, based on some specialty literature references, but also with proper observations. Mathematical model were elaborated for position, speeds and accelerations determination, for some interest points, used for experimental modeling, according with a new prostheses and orthotics design used for children gait rehabilitation.

Based on the experimental analysis, by using CONTEMPLAS Motion equipment, the experimental data validate the analytical ones obtained in the human lower limb kinematic analysis frame. On x-axis the M point component has an appropriate form (see figure 2 and figure 8). The data limit is between 110-380milimeters in the real time and in case of the computed data the limit is between 100-392milimeters and this is almost appropriate as time and values. It seems that the 4 years old healthy child performs a 270 millimeters as a gait distance.

In another order on y-axis direction the M point component also has an appropriate form (see figure 3 and figure 9). The data limit on this direction in real time is between -355 to -330 millimeters. As an observation the negative values in the real time was influenced by the displacement mode of the CONTEMPLAS reference coordinate system. In the case of the computed data the limit is between 6-32 millimeters. These certify that the computed data are almost appropriate with the real ones obtained on experimental way and also validate the kinematic method onto human lower limb studies.

6. Acknowledgements

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